Explanation of Robert Resnick's Derivation of Lorentz Transformation Formulae in the Special Theory of Relativity

by

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Robert Resnick, in his book 'Introduction to Special Relativity', writes the following in his derivation of the Lorentz Transformation formulae:

"Let us assume that at time $t = 0$ a spherical electromagnetic wave leaves the origin of [the stationary reference frame] S, which coincides with the origin of [the moving reference frame] S' [(moving with a speed v along the x-axis of the frame S)] at that moment. The wave propagates with a speed c in all directions in each inertial frame. Its progress, then, is described by the equation of sphere whose radius expands with time at rate c in terms of either the primed or unprimed set of coordinates. That is,

$$x^2 + y^2 + z^2 = c^2t^2 \ldots (2\text{-}4)$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2 \ldots (2\text{-}5)"$$

This requires an explanation which will be evident from the following question which raises a confusion and the answer which answers it.

Q. In Robert Resnick's derivation of the Lorentz transformation equations (vide Robert Resnick, "Introduction to Special Relativity"), he makes the mistake of not taking into account the translation vt' in time t' of the moving frame of reference S'. We have to note that the wave which radiated from the origin of the stationary frame S is not a sphere of radius ct' and centre at the origin of S' at time t', but with radius ct' and centre at $(-vt', 0, 0)$. Actually, the observer in the S' frame thinks that he is at rest and that the frame S is moving with a velocity v in the direction opposite to that of the velocity v of the S' frame. The following figure (Fig. 1) illustrates this fact.

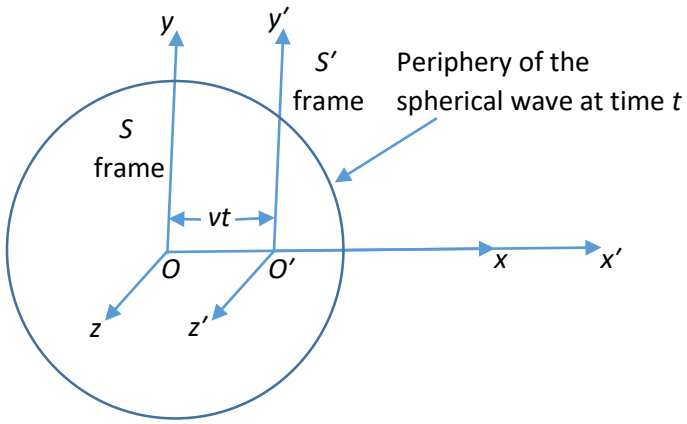

Fig. 1: Diagram Depicting the Conditions of the S Frame, S' Frame and the Radiated Spherical Electromagnetic Wave at Time t (The distance between the S and the S' frames appears to be vt when seen from the S frame and vt' when seen from the S' frame.)

If this fact is taken into account, the 2 equations written by observers in the S and the S' frames will respectively be

$$x^2 + y^2 + z^2 = c^2t^2 \ldots (1)$$
$$(x' + vt')^2 + y'^2 + z'^2 = c^2t'^2 \ldots (2)$$

Ans.: Resnick says in his book that the velocity of light is constant irrespective of the velocity of the frame of reference. That means that the velocity of light is the same, i.e., c, irrespective of whether we see the light from the stationary S frame or the moving S' frame. That means that the periphery of the light wave which radiated from the origin of the S frame at time $t = 0$ when S and S' frames were coincident will appear to be a sphere both from the S frame and the S' frame, but with centre at the origin of the S frame and radius ct when viewed from the S frame and centre at the origin of the S' frame and radius ct' when viewed from the S' frame. When viewed from the S' frame, it will not appear that the periphery of the light wave is a sphere with centre at ($-vt'$, 0, 0) and radius ct'. How the spherical wave will actually appear from the S and the S' frames at time t is shown in the following figure (Fig. 2).

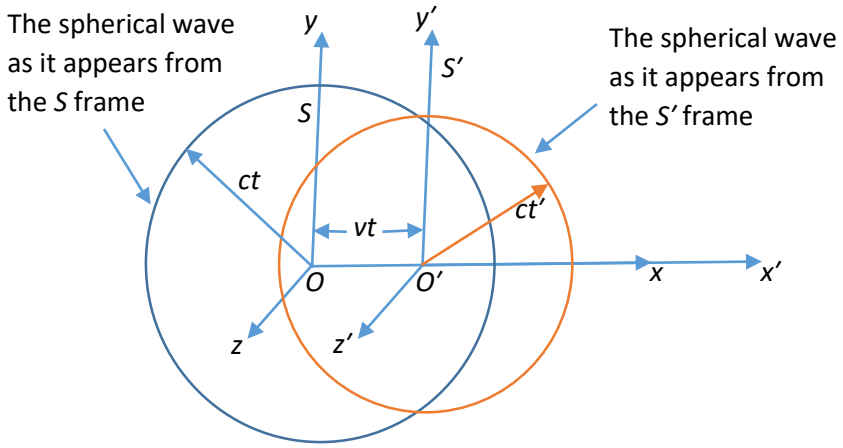

Fig. 2: Diagram Depicting How the Spherical Wave Will Actually Appear from the S and the S′ Frames at time t (The distance between the S and the S' frames appears to be vt when seen from the S frame and vt' when seen from the S' frame.)

Then, the equations of the light wave when it is viewed from the S frame and the S' frame will respectively be

$x^2 + y^2 + z^2 = c^2t^2 \ldots (1)$, and

$x'^2 + y'^2 + z'^2 = c^2t'^2 \ldots (2)$

This is the explanation behind equations (2-4) and (2-5) given by Robert Resnick in his derivation of the Lorentz Transformation formulae in his book.